Non-local Momentum Transport Parameterizations

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Outline

- Historical view: gravity wave drag (GWD) and convective momentum transport (CMT)
- GWD development
  - -semi-linear theory
  - -impact
- CMT development
  - -theory
  - -impact

Both parameterizations of recent vintage compared to radiation or PBL

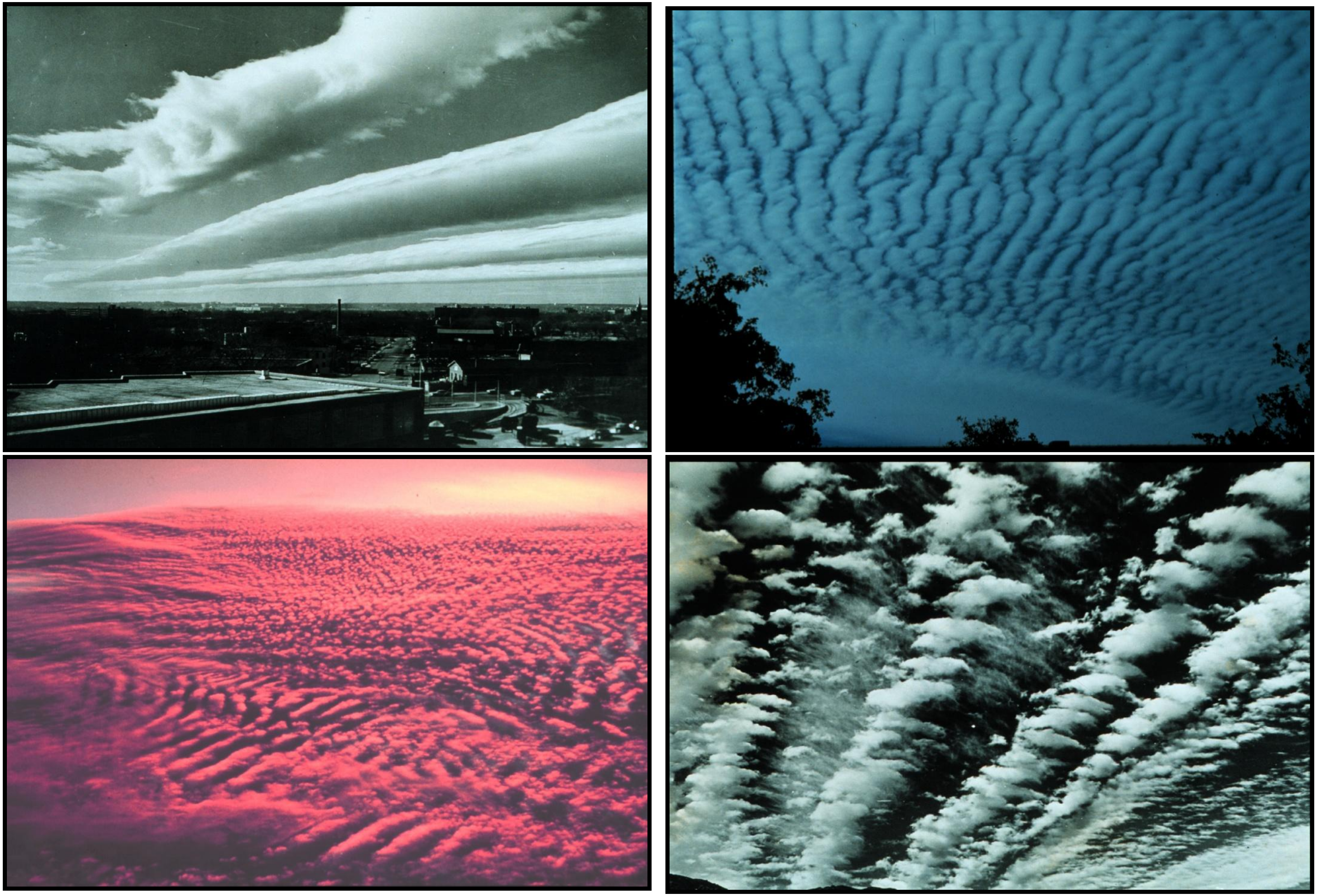
GWD

- 1960's discussion by Philips, Blumen and Bretherton
- 1970's quantification Lilly and momentum budget by Swinbank
- 1980's incorporation into NWP and climate models-Miller and Palmer and McFarlane

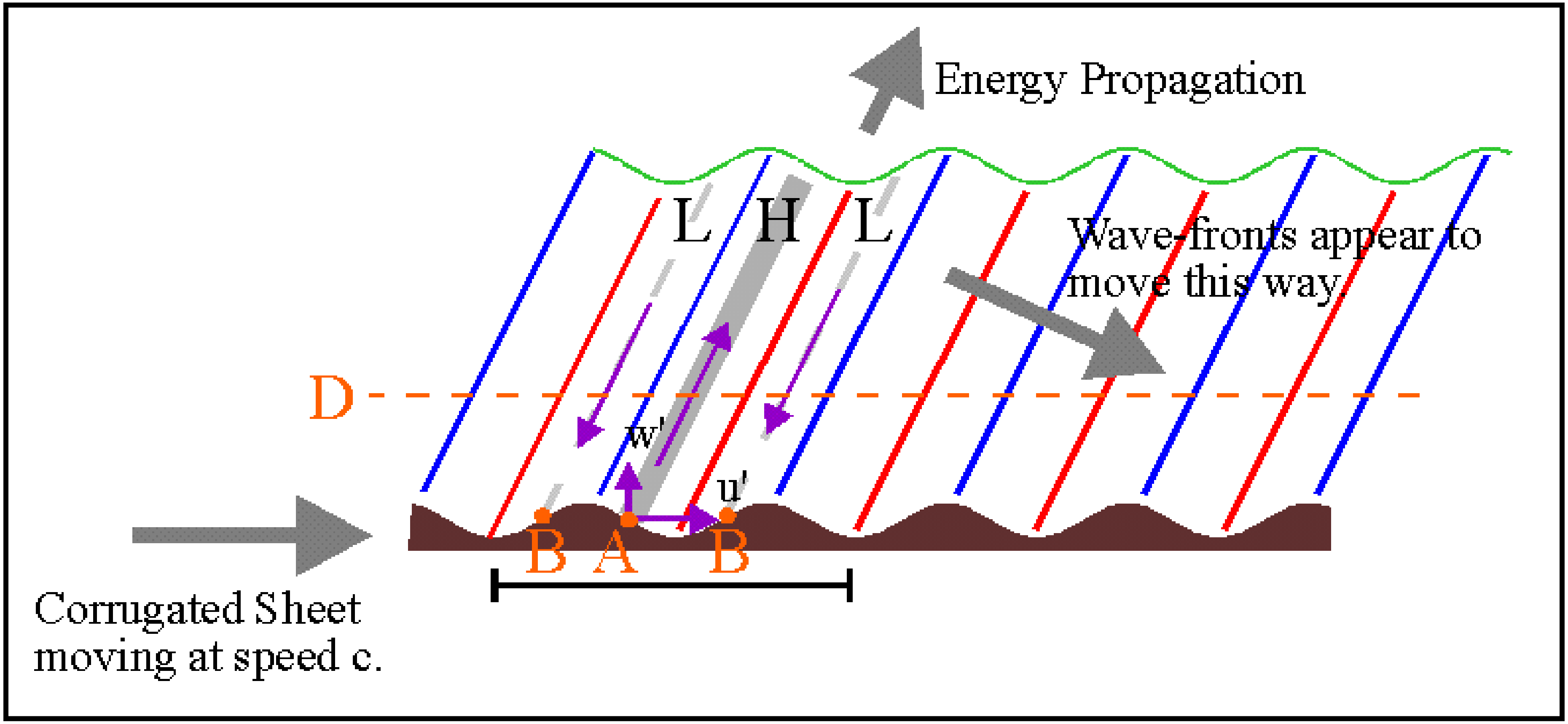
CMT

- 1972 cumulus vorticity damping 'observed' Holton
- 1976 Schneider and Lindzen -Cumulus Friction
- 1980's NASA GLAS model-Helfand
- 1990's pressure term-Gregory

Atmospheric Gravity Waves



Simple gravity wave model



Topographic Gravity Waves and Drag

- Flow over topography generates gravity (i.e. buoyancy) waves
- <u’w’> is positive in example
- Power spectrum of Earth’s topography α k^{-2} so there is a lot of subgrid orography
- Subgrid orography generating unresolved gravity waves can transport momentum vertically
- Let’s parameterize this mechanism!

Begin with linear wave theory

Simplest model for gravity waves:

$$\frac{\partial w'}{\partial t} + u_0 \frac{\partial w'}{\partial x} + \frac{1}{\rho_0}\frac{\partial p'}{\partial z} - \frac{\rho'}{\rho} g = 0$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0$$

$$\frac{\partial \theta'}{\partial t} + u_0 \frac{\partial \theta'}{\partial x} + w' \frac{\partial \theta}{\partial z} = 0$$

with

$$\frac{\rho'}{\rho_0} = \frac{\theta'}{\theta_0}$$

$$\left(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}\right)^2 \left(\frac{\partial^2 w'}{\partial x^2} + \frac{\partial^2 w'}{\partial z^2}\right) + N^2 \frac{\partial^2 w'}{\partial x^2} = 0$$

Assume w’ α $e^{i(kx+mz-\sigma t)}$ gives the dispersion relation

$$(\sigma - u_0 k)^2 (k^2 + m^2) - N^2 k^2 = 0$$

or

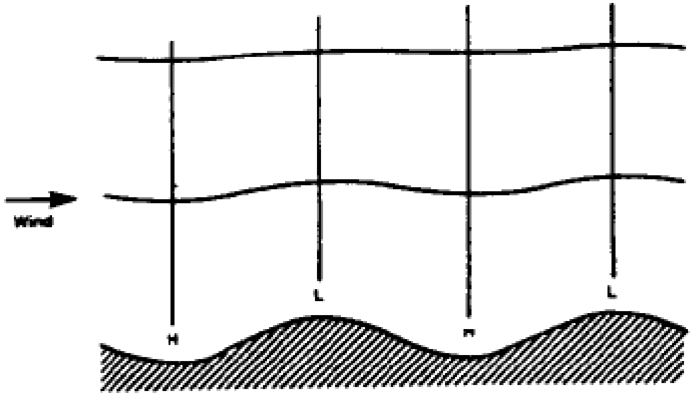
$$\hat{\sigma} = \sigma - u_0 k = \pm \frac{Nk}{\sqrt{k^2 + m^2}},$$

Linear theory (cont.)

Sinusoidal topography ; set σ=0.

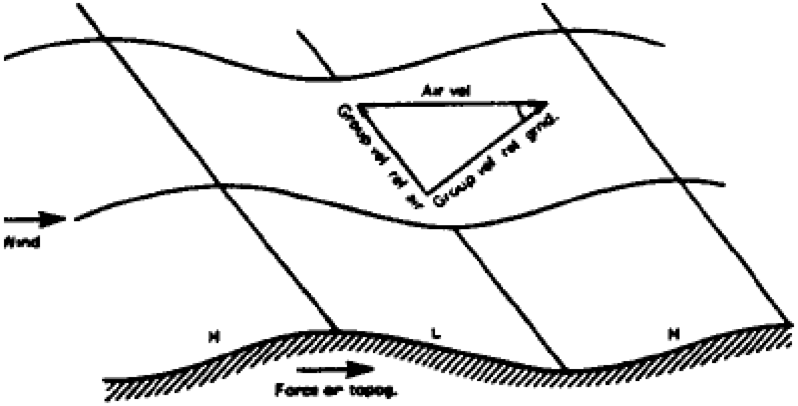
$h = h_{\mathrm{m}} \sin kx$ Gives linear lower BC $w|_{z=0} = u_0 \frac{\partial h}{\partial x} = u_0 k h_{\mathrm{m}} \cos kx$

Small scale waves
$k > N/U_0$ decay



(a)

Larger scale waves
$k < N/U_0$ propagate



(b)

Semi-linear Parameterization

Propagating solution with upward group velocity

$$w' = u_0 k h_m \cos(kx + mz)$$

$$u' = -u_0 m h_m \cos(kx + mz)$$

$$\rho_0 \overline{u'w'} = -\frac{1}{2} u_0^2 \rho_0 k m h_m^2$$

In the hydrostatic limit

$$m = \frac{N}{u_0}$$

$$\rho_0 \overline{u'w'} = -\frac{1}{2} u_0 \rho_0 k N h_m^2$$

The surface drag can be related to the momentum transport

$$D = \int_0^{2\pi/k} p'(x,h) \frac{\partial h}{\partial x} dx$$

$$= \int_0^{2\pi/k} p'(x,0) \frac{\partial h}{\partial x} dx$$

$$D = -\int_0^{2\pi/k} \rho_0 u'w'|_{z=0} dx$$

Momentum transport invariant by Eliassen-Palm. Deposited when linear theory is invalid (CL, breaking)

$$N^2{}_{\text{total}} = N^2 \left\{ 1 + \left(\frac{N \delta h}{u_0} \right) \cos\phi \right\}$$

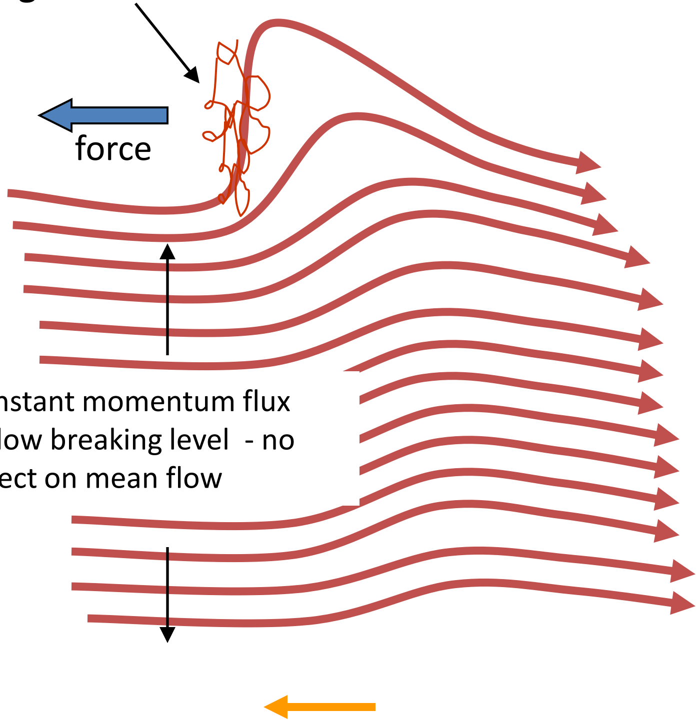
$$\eta_{\text{total}} = \eta \left\{ 1 + Ri^{1/2} \left(\frac{N \delta h}{u_0} \right) \cos\phi \right\}$$

δh=isentropic displacement

η=U_z-

ϕ=phase

Gravity Wave Drag Parameterization



Convective or shear instabilty begins to dissipate wave- ***momentum flux no longer constant***

force

constant momentum flux below breaking level - no effect on mean flow

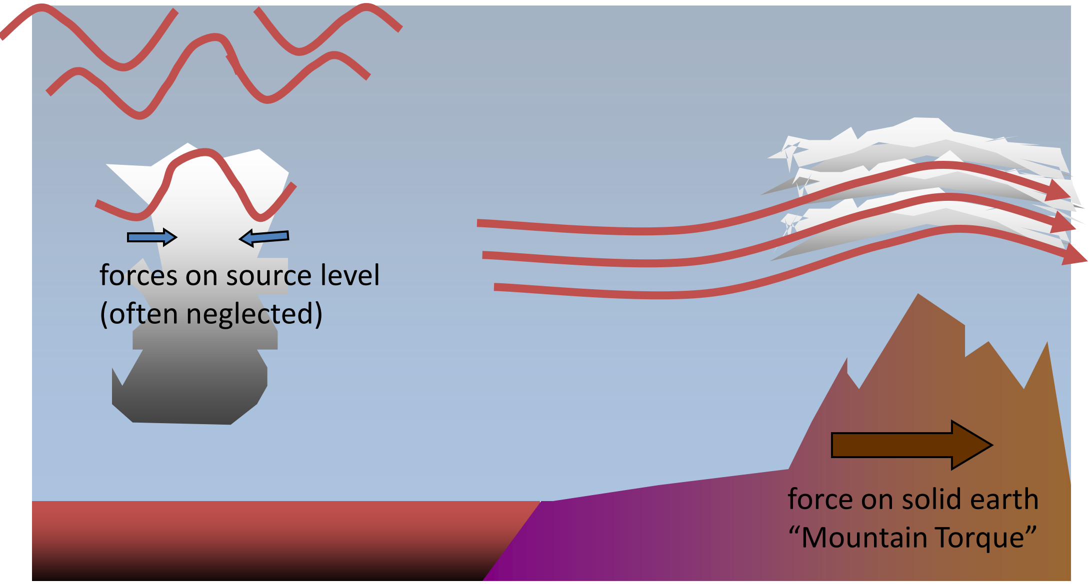
wave phase speed relative to flow

Waves propagate vertically, amplitude grows as $\rho^{-1/2}$ (energy cons.). Eventually waves induce unstable flow situation. Amplitude is assumed to remain exactly critical from there on. This leads to momentum flux ***divergence*** and wind tendency:

$$\partial_t[u] \sim -\frac{1}{\rho}\partial_z \rho[u'w'] \sim -\hat{U}\hat{W}\frac{1}{\rho}\partial_z \rho$$

Conceptual Model: 2D, linear, WKB wave model. Forcing by subgrid variance in topography, heating amplitudes

CAM “Physics” - Gravity Wave Drag

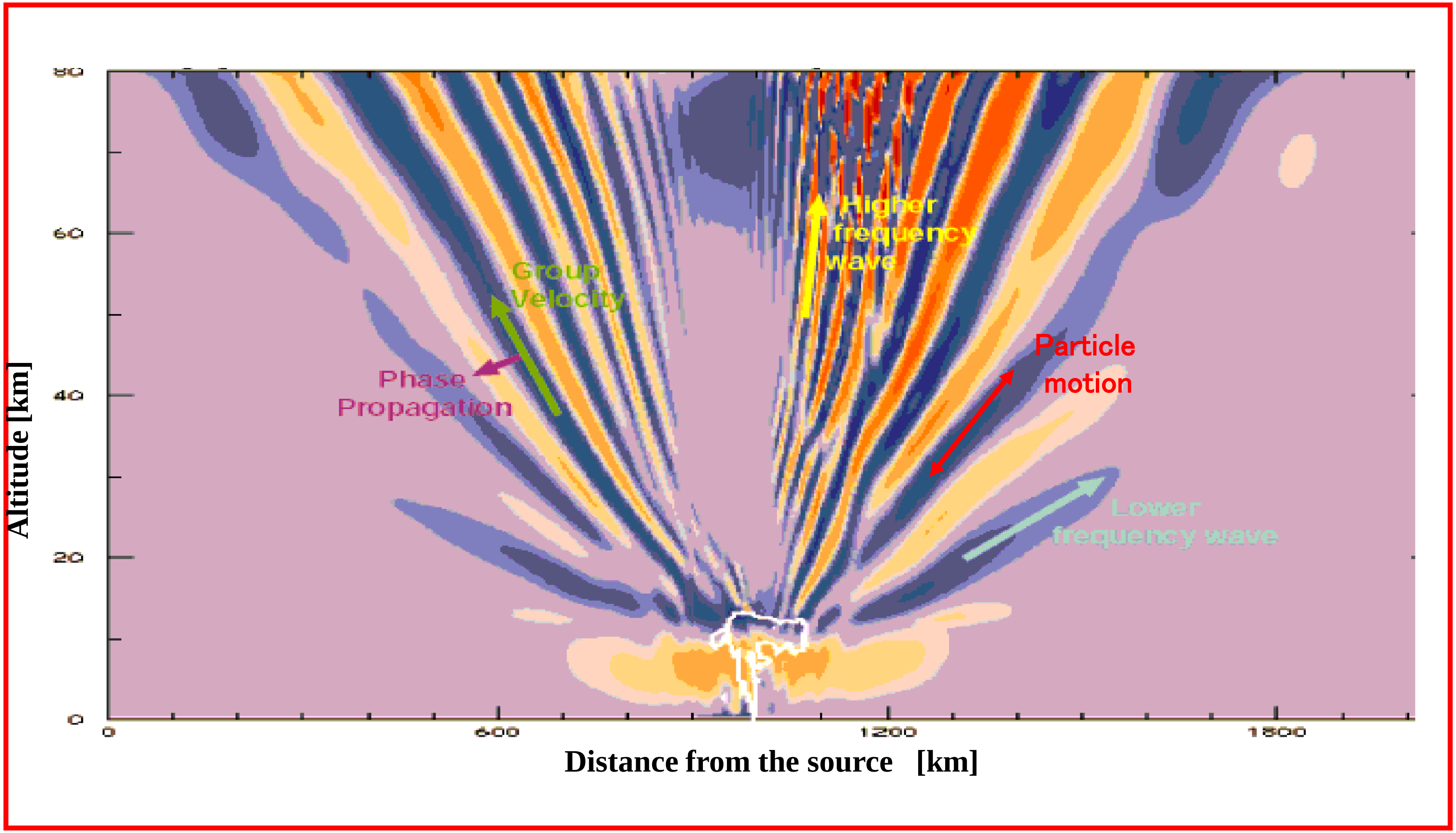


Gravity Wave Sources generally located in troposphere.

In nature, sources include convection and fronts in addition to flow over mountains.

Current parameterization includes orographic source plus spectrum of non-zero phase speed waves. Horizontal scales of GW span 1000s of km (resolved) to several km (need to be parameterized). CAM_future will prognose convective and frontal sources

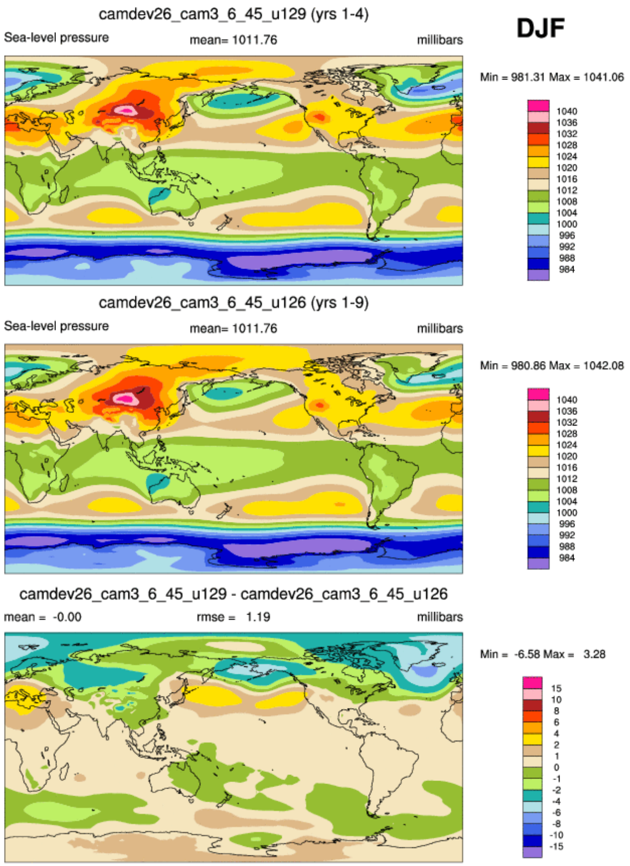
Propagation of AGW



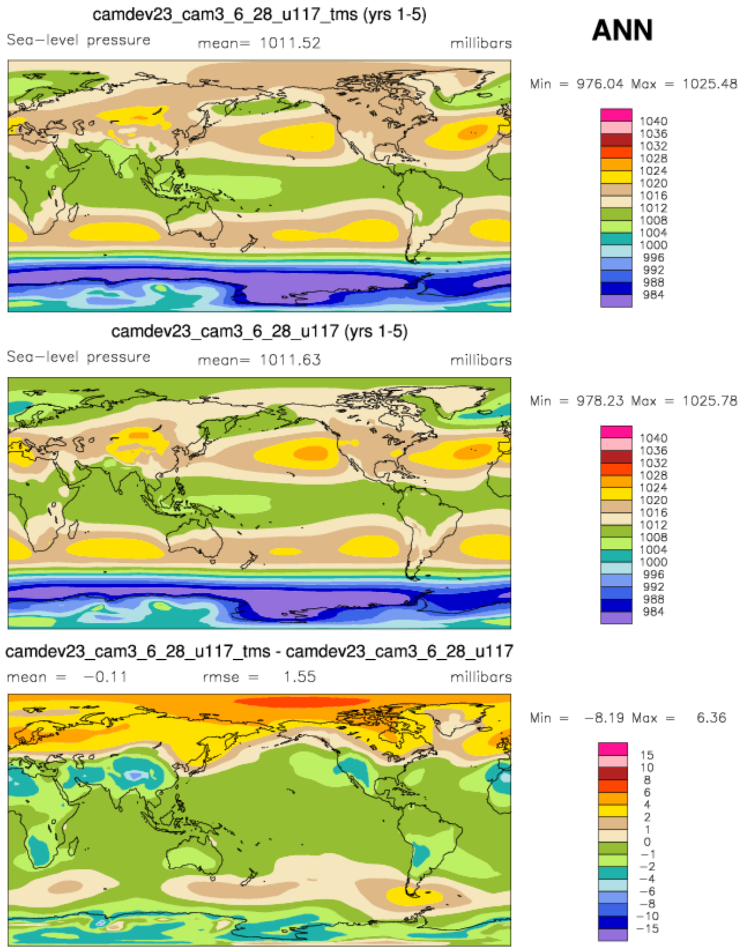
Alexander 2002 - CEDAR

Impact of changing:

critical Froude number



turbulent mountain stress : $z_0(h)$



GWD summary

- Simple parameterization built out of linear theory
- Extensible to more elaborate non-linear cases; e.g. Lott and Miller blocking effects and orographic-flow alignment
- CAM code modules gw_drag.F90 and trb_mtn_stress.F90
- Can change surface winds directly and indirectly

CMT rationale

- In cumulus towers updraft and downdraft transport constituents in the vertical
- Reynolds’ stresses like <u’w’> can lead to substantial momentum transfer between PBL and cloud top
- Cumulus parameterization already uses computes vertical transfer of constituents like q and h
- Use this to parameterize CMT

Convective Momentum Transport

$$\frac{D\mathbf{U}}{Dt} + \dots = \mathbf{F}_c$$

'Cumulus Friction'

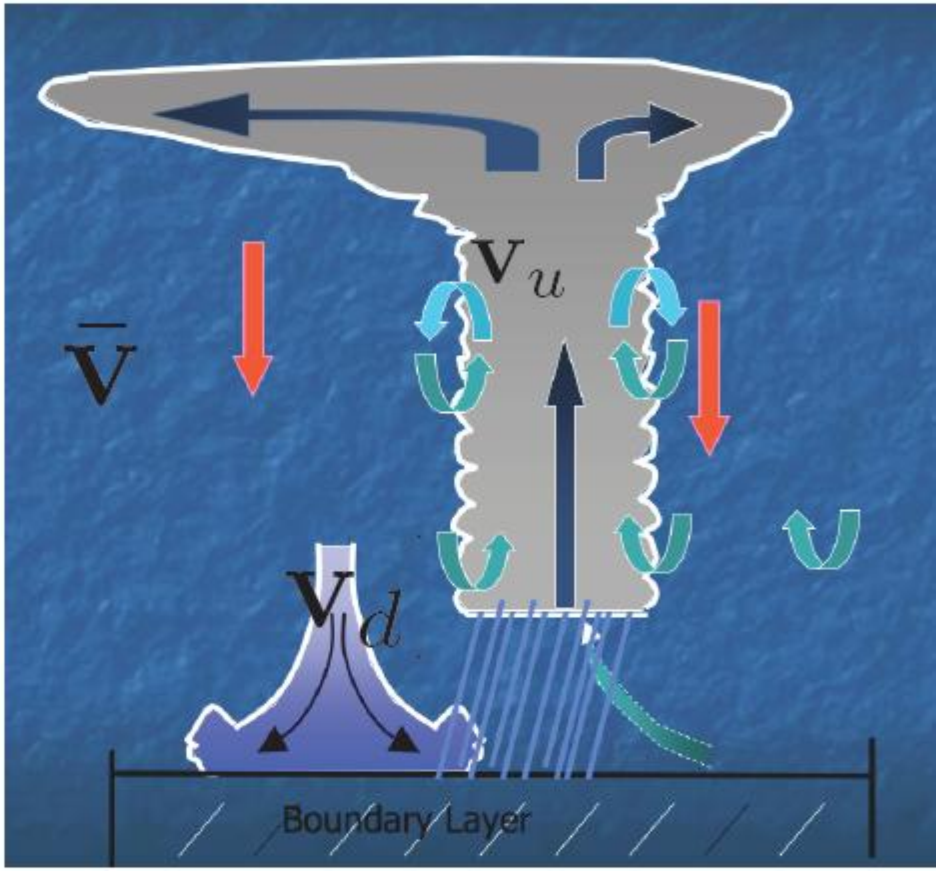
$$\mathbf{F}_c = -\frac{\partial}{\partial p}\overline{\mathbf{v}'\omega'}$$

$$= \frac{\partial}{\partial p} (M_u ((\mathbf{v}_u - \bar{\mathbf{v}}) + M_d (\mathbf{v}_d - \bar{\mathbf{v}}))$$

Updraft Mass Flux

Updraft Velocity

Environmental Velocity



Are we forgetting about something?

MOMENTUM NOT A LAGRANGIAN CONSERVED QUANTITY : NEED TO ACOUNT FOR PRESSURE GRADIENT

In-Cloud Velocities

Schneider and Lindzen (1976)

$$-\frac{\partial(M_u\,\mathbf{v}_u)}{\partial p} = E_u\,\bar{\mathbf{v}} - D_u\,\mathbf{v}_u$$

$$-\frac{\partial(M_d\,\mathbf{v}_d)}{\partial p} = E_d\,\bar{\mathbf{v}}$$

assumes that in-cloud velocities are dependent ONLY on lateral entrainment and detrainment rates

Entrainment

Detrainment

Zhang and Cho (1991)

Gregory et al (1997)

$$-\frac{\partial(M_u\,\mathbf{v}_u)}{\partial p} = E_u\,\bar{\mathbf{v}} - D_u\,\mathbf{v}_u + \mathbf{P}_G^u$$

$$-\frac{\partial(M_d\,\mathbf{v}_d)}{\partial p} = E_d\,\bar{\mathbf{v}} + \mathbf{P}_G^d$$

account for the pressure gradient term

Gregory et al: (1997)

$$\mathbf{P}_G^u = -C_u\,M_u\,\frac{\partial\bar{\mathbf{v}}}{\partial p}$$

$$\mathbf{P}_G^d = -C_d\,M_d\,\frac{\partial\bar{\mathbf{v}}}{\partial p}$$

How do you get this expression for the pressure gradient term?

From the anelastic pressure equation:

$$\begin{aligned}\nabla^2 p = \quad & \boldsymbol{\nabla}\left\{-2\rho\left[\frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y}\right]\right. \\ & - \rho\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] \\ & \left. + w^2\rho\frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial \rho}{\partial z}\right)\right\},\end{aligned}$$

Linearize the RHS to get in x-z plane:

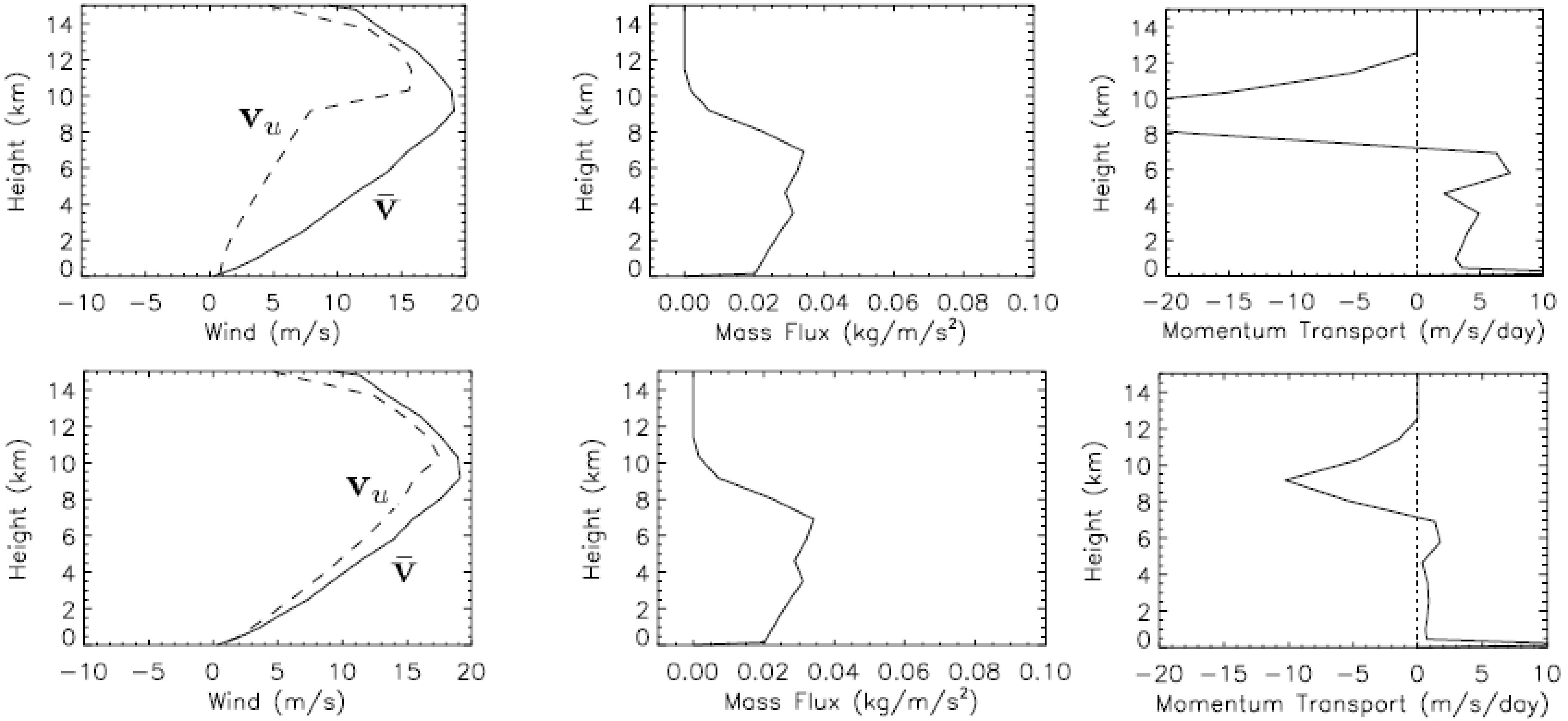
$$\nabla^2\left(\frac{\partial p}{\partial x}\right) = \frac{\partial}{\partial x}\left\{-2\rho\frac{\partial w}{\partial x}\frac{\partial \overline{u}}{\partial z}\right\}.$$

Lastly, assume sinusoidal form in x and z for w and p.
C’s are tuning coefficients for these sinusoidal scales

SCAM Example

Schneider and Lindzen (1976)

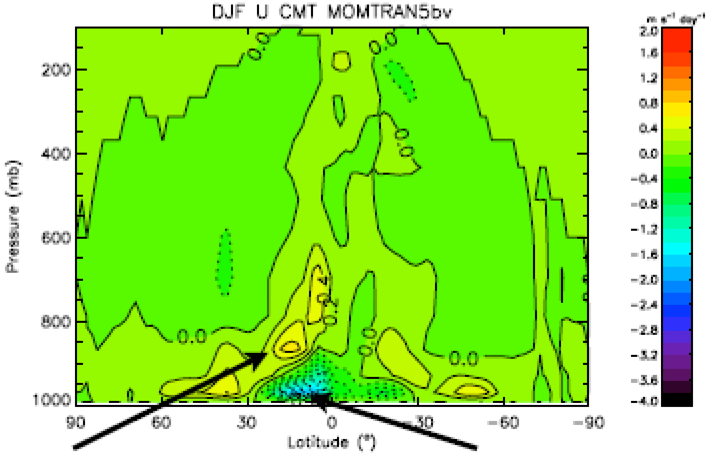
$$\frac{\partial}{\partial p}\left(M_u\left((\mathbf{v}_u - \bar{\mathbf{v}}\right)\right.$$



Gregory et al (1997)

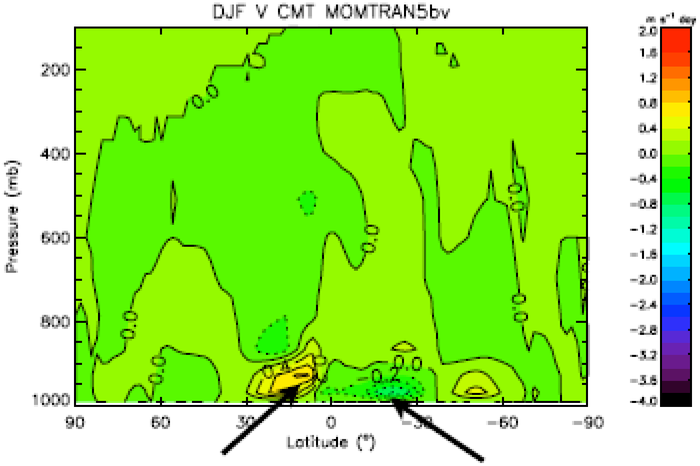
Decreased CMT

Gregory et al (1997) in CAM3:



0.8 m/s/day

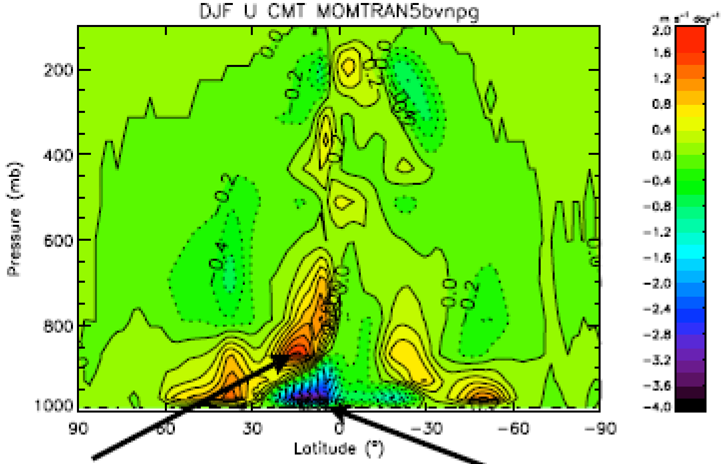
-1.4 m/s/day



1.2 m/s/day

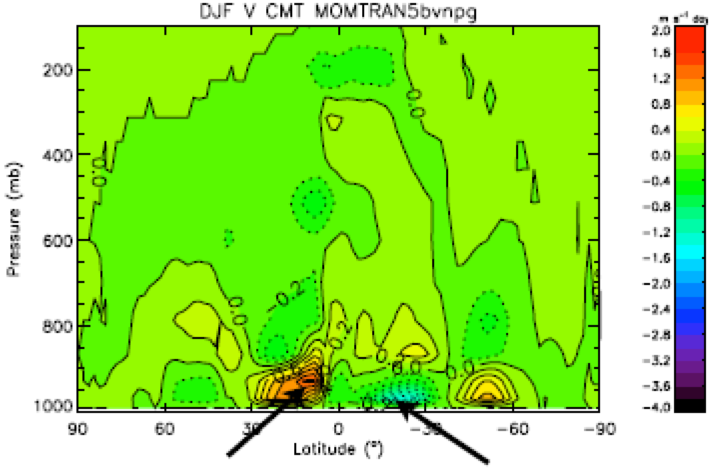
-0.8 m/s/day

Schneider & Lindzen (1976) in CAM3:



2.0 m/s/day

-3.5 m/s/day



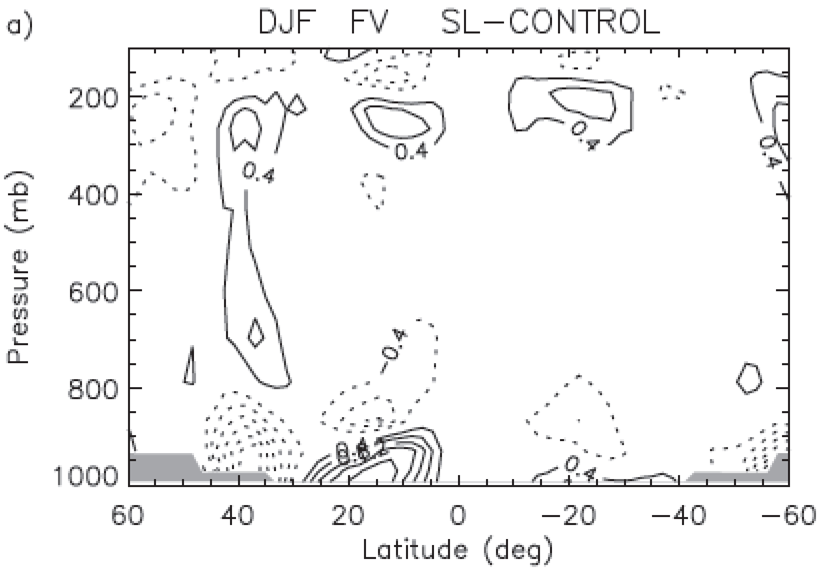
2.0 m/s/day

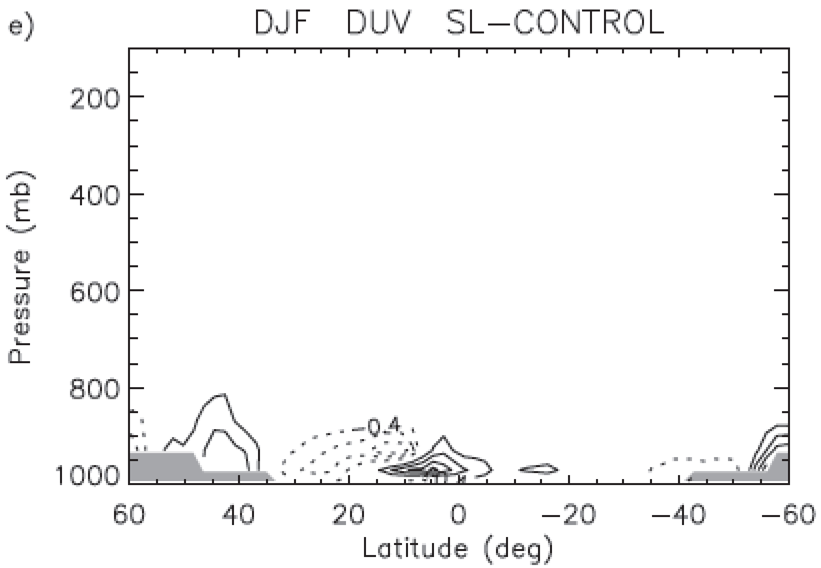
-1.2 m/s/day

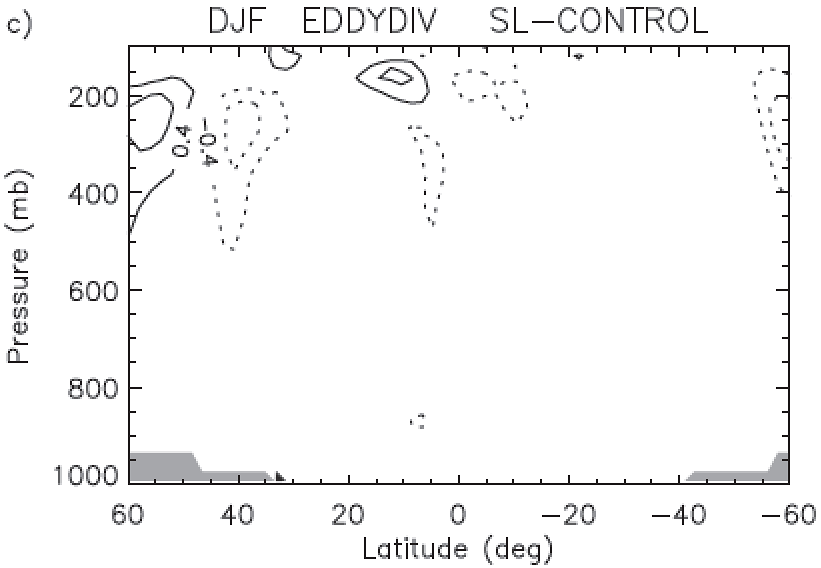
How does CMT influence climate?

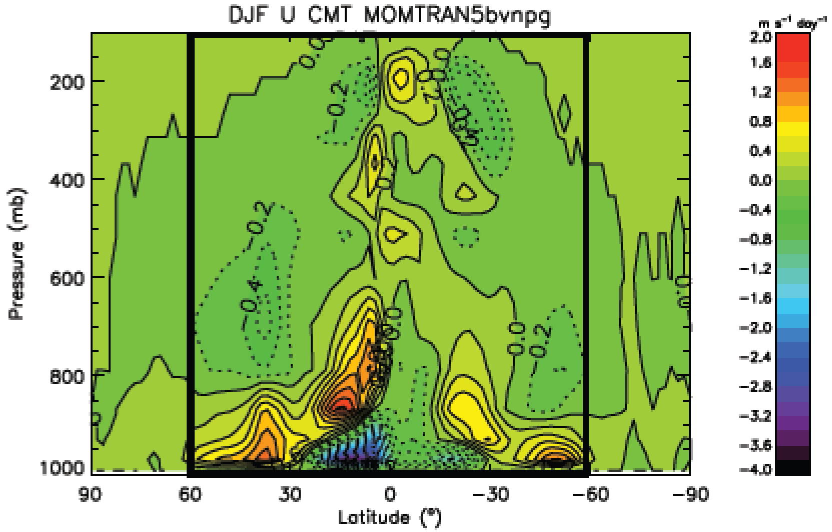
$$\begin{aligned}\frac{\partial[u]}{\partial t} = \overset{\substack{\text{Coriolis}\\ \text{Torque}}}{\overset{\downarrow}{f\,[v]}} & \boxed{-\frac{1}{a^2\,cos^2\phi}\,\frac{\partial}{\partial\phi}\left(a\,cos^2\phi\,[v]\,[u]\right) - \frac{\partial}{\partial p}\left([\omega][u]\right)} \leftarrow \text{Mean Divergence}\\ & \boxed{-\frac{1}{a^2\,cos^2\phi}\,\frac{\partial}{\partial\phi}\left(a\,cos^2\phi\,[v^*u^*]\right) - \frac{\partial}{\partial p}\left([\omega^*u^*]\right)} \leftarrow \text{Eddy Divergence}\\ & \qquad\qquad + \underset{\substack{\uparrow\\ \text{GW}\\ \text{Drag}}}{[X_{GW}]} + \underset{\substack{\uparrow\\ \text{CMT}}}{[F_{cx}]} + \underset{\substack{\uparrow\\ \text{Vertical}\\ \text{Diffusion}}}{[D_u]}\end{aligned}$$

Details in Richter and Rasch (2007)



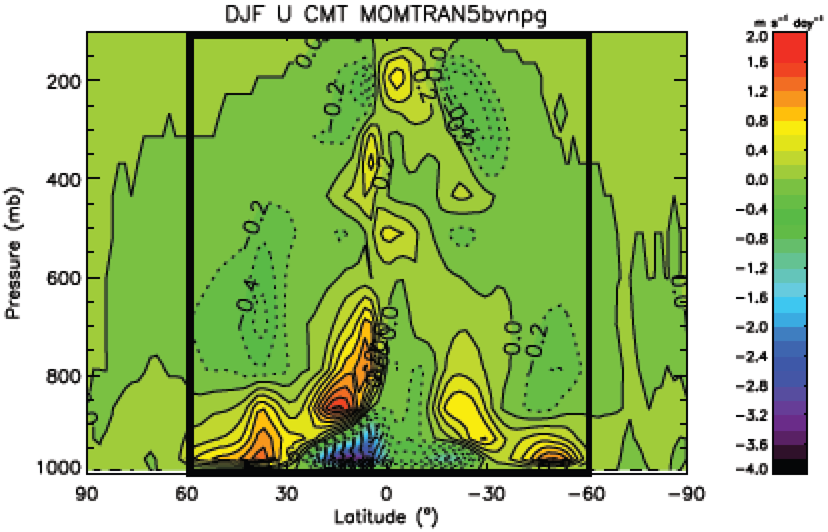




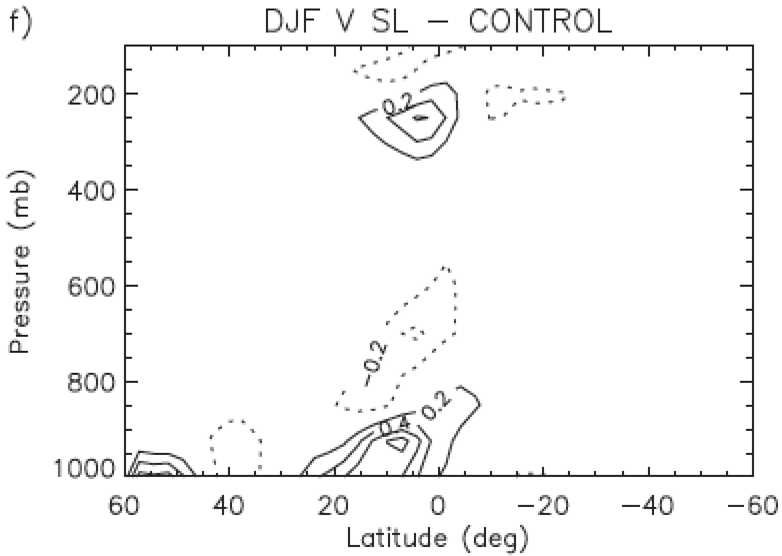


SL - Control:

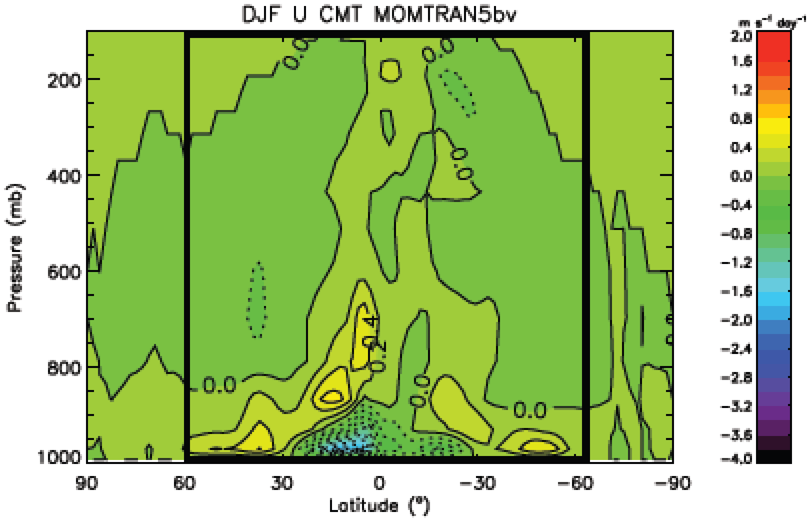
$$f\triangle[v] \propto -\triangle[F_{cx}]$$



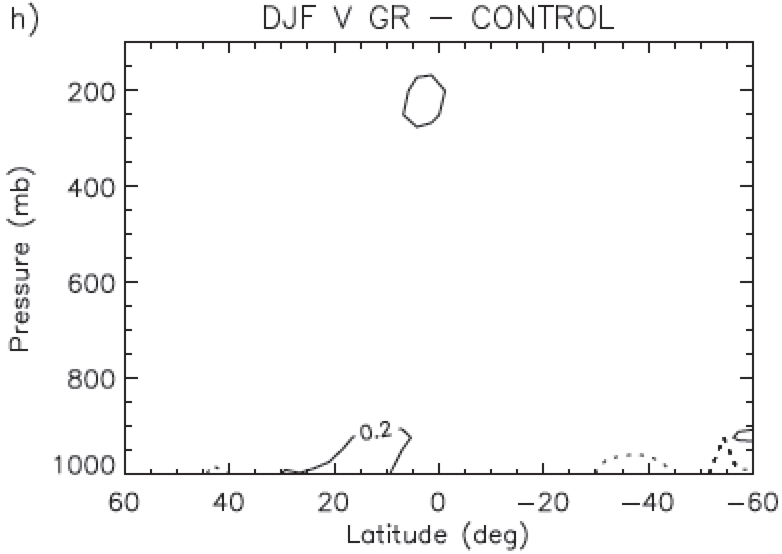
DJF U CMT MOMTRAN5bvnpg



f) DJF V SL − CONTROL



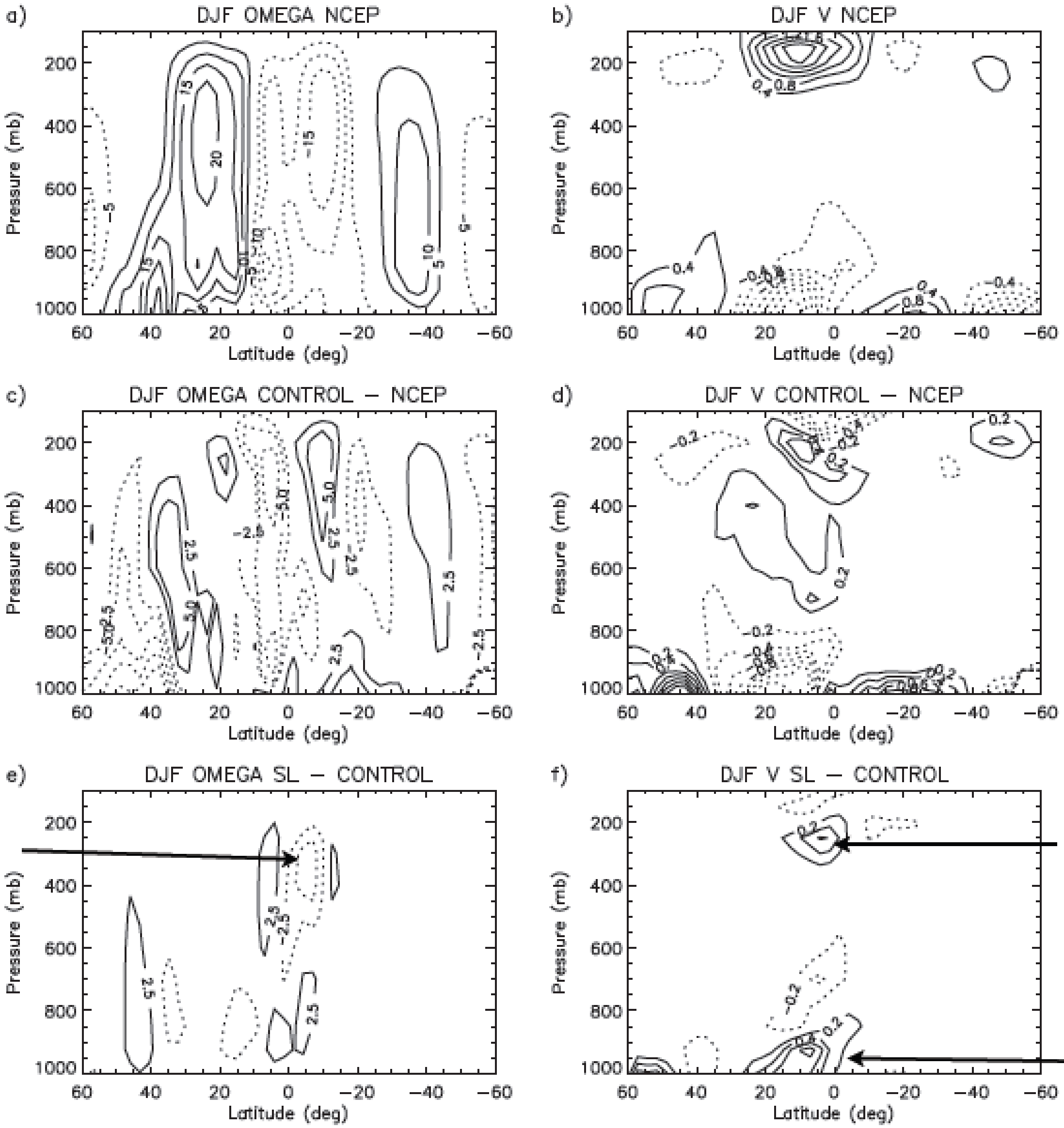
DJF U CMT MOMTRAN5bv



h) DJF V GR − CONTROL

$$f\triangle[v] \propto -\triangle[F_{cx}]$$

Hadley Circulation

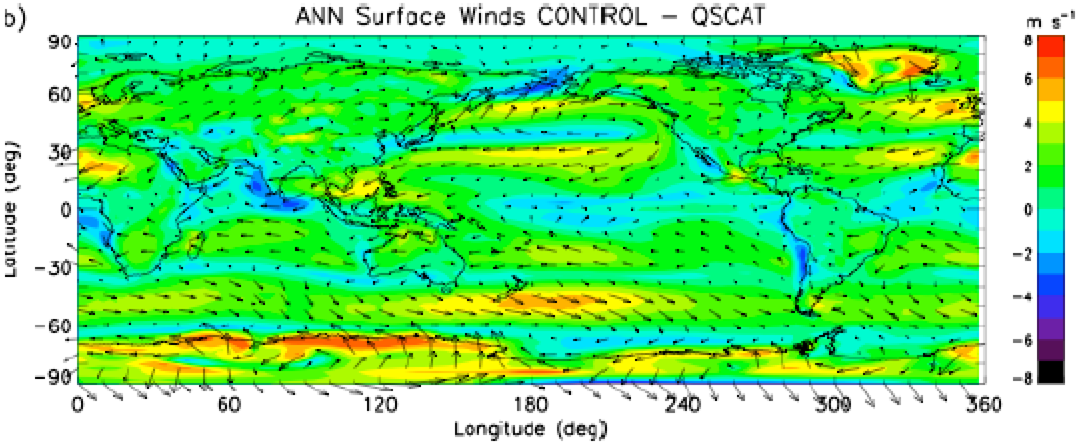


Strengthening

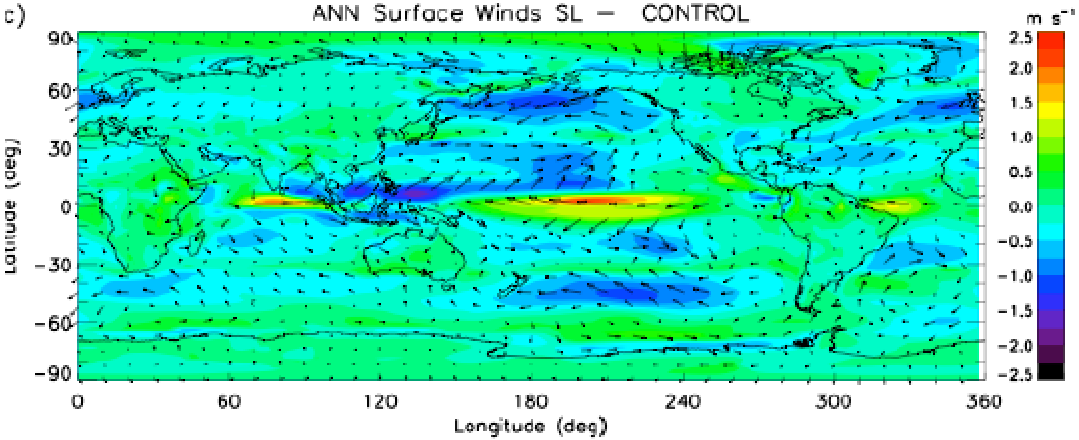
Strengthening

Weakening

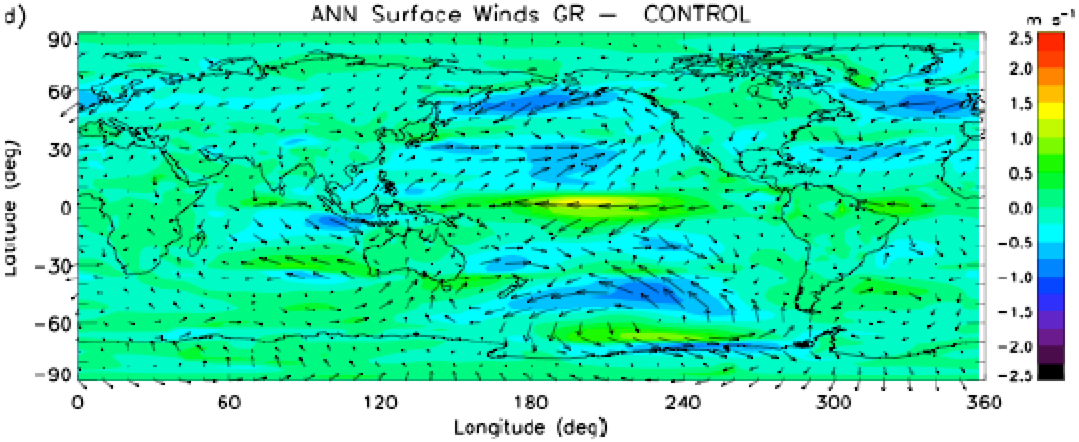
Control - OBS



Schneider & Lindzen (1976)-control



Gregory et al (1997) -control

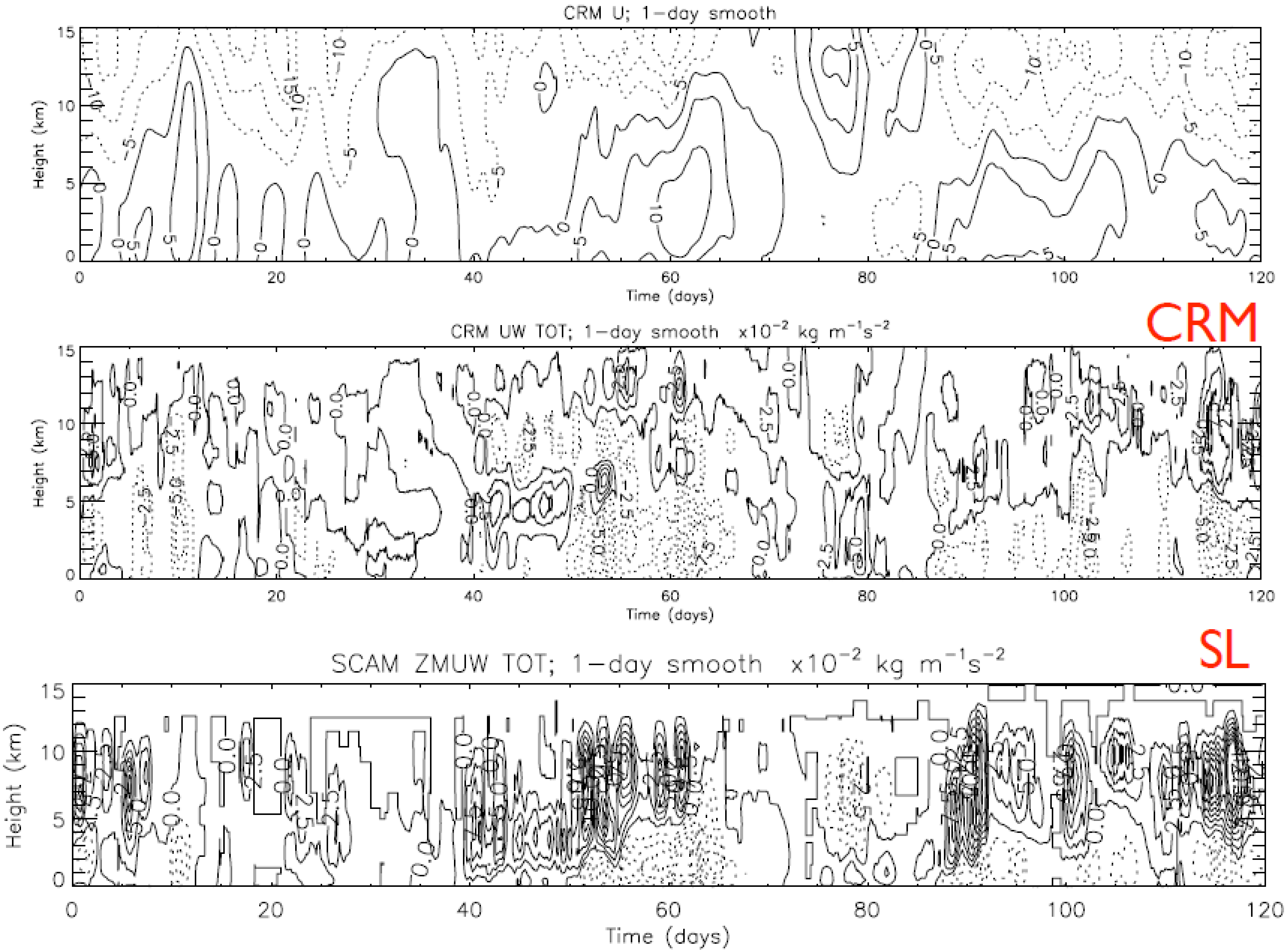


What about the tuning coefficient?

$$\mathbf{P}_G^u = -\boxed{C_u} M_u \frac{\partial \bar{\mathbf{v}}}{\partial p}$$

$$\mathbf{P}_G^d = -\boxed{C_d} M_d \frac{\partial \bar{\mathbf{v}}}{\partial p}$$

- SCAM vs TOGA COARE CRM simulation comparison (collaboration with Chris Bretherton)
- CRM: 250 x 250 km; dx = dy = 1km
- forced by SST's, large scale vertical velocity, and horizontal advection



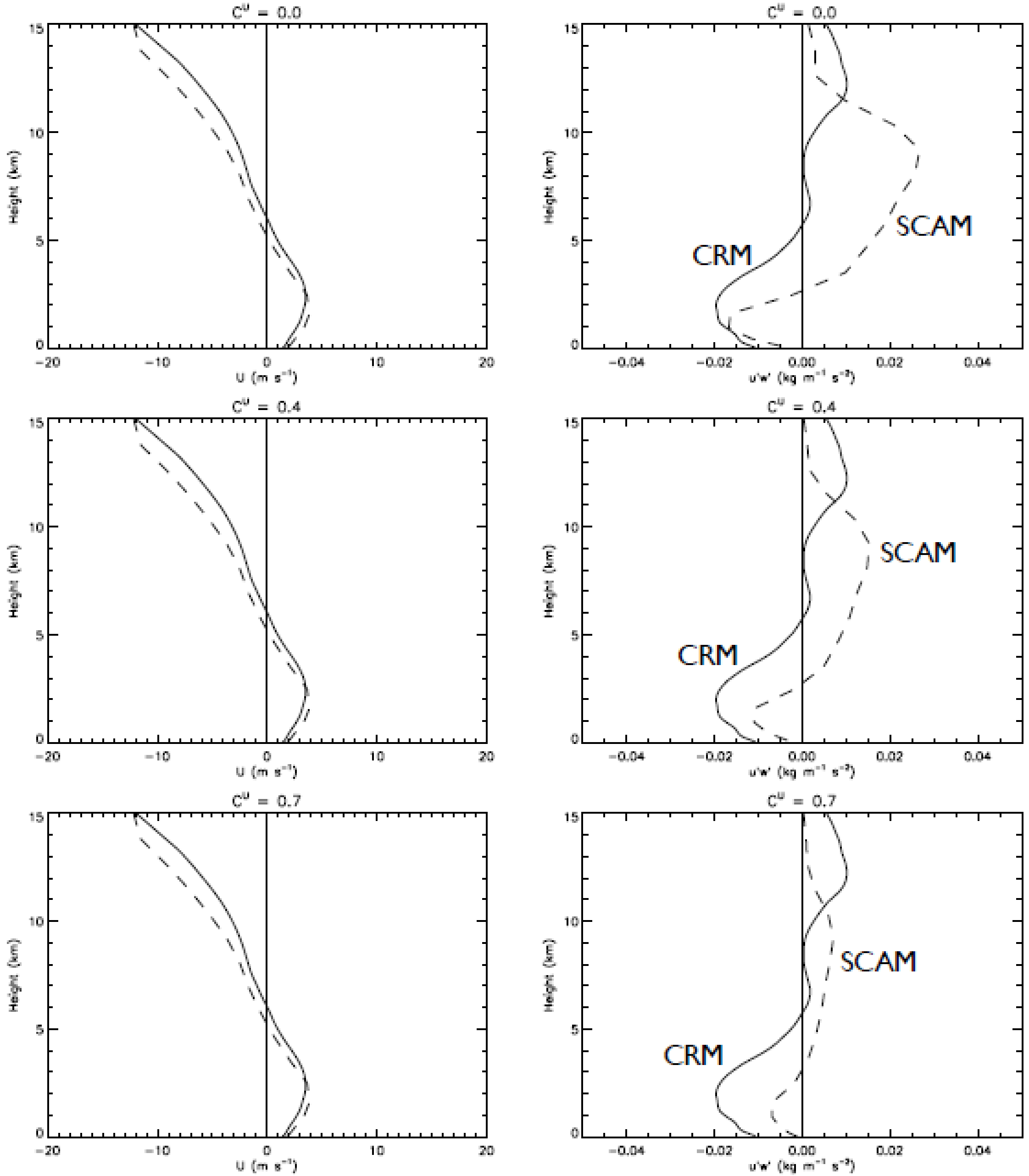
CRM U; 1-day smooth

CRM UW TOT; 1-day smooth x10^{-2} kg m^{-1}s^{-2}

SCAM ZMUW TOT; 1-day smooth x10^{-2} kg m^{-1}s^{-2}

CRM

SL



SL

Cu = 0.0

Cu = 0.4

GR

Cu = 0.7

CMT summary

- Makes significant change in tropical circulation and convection
- Makes use of linearized theory
- Also used high resolution process model and single column model (SCAM) to refine parameterization
- Cu=0.4 gives best fit in SCAM tests
- In module zm_conv.F90